# Design of PI and PID Controllers With Transient Performance Specification

J. C. Basilio and S. R. Matos

***Abstract*—Proportional-integral-derivative (PID) controllers are widely used in industrial control systems because of the reduced number of parameters to be tuned. The most popular design technique is the Ziegler–Nichols method, which relies solely on parameters obtained from the plant step response. However, besides being suitable only for systems with monotonic step response, the compensated systems whose controllers are tuned in accordance with the Ziegler–Nichols method have generally a step response with a high-percent overshoot. In this paper, tuning methods for proportional-integral (PI) and PID controllers are proposed that, like the Ziegler–Nichols method, need only parameters obtained from the plant step response. The methodology also encompasses the design of PID controllers for plants with underdamped step response and provides the means for a systematic adjustment of the controller gain in order to meet transient performance specifications. In addition, since all the development of the methodology relies solely on concepts introduced in a frequency-domain-based control course, the paper has also a didactic contribution.**

***Index Terms*—Control education, control system design, process control, proportional-integral-derivative (PID) controllers, root-locus diagram.**

## I. Introduction

PROPORTIONAL-integral-derivative (PID) controllers [1]–[3] are widely used in industrial control systems because of the reduced number of parameters to be tuned. They provide control signals that are proportional to the error between the reference signal and the actual output (proportional action), to the integral of the error (integral action), and to the derivative of the error (derivative action), namely

$$u(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_0^t e(\tau)\, d\tau + T_d \frac{d}{dt} e(t) \right] \quad (1)$$

where $u(t)$ and $e(t)$ denote the control and the error signals, respectively, and $K_p, T_i$, and $T_d$ are the parameters to be tuned. The corresponding transfer function is given as

$$K(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right). \quad (2)$$

The main features of PID controllers are the capacity to eliminate steady-state error of the response to a step reference signal (because of integral action) and the ability to anticipate output changes (when derivative action is employed).

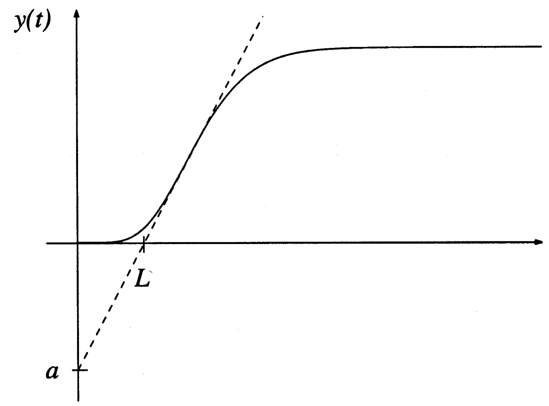


Fig. 1. Step response for the tuning of PID controllers according to Ziegler–Nichols method.

TABLE I
TUNING OF PID CONTROLLER PARAMETERS ACCORDING TO ZIEGLER–NICHOLS METHOD

| Controller | $K_p$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $1/a$ | | |
| PI | $0.9/a$ | $3L$ | |
| PID | $1.2/a$ | $2L$ | $L/2$ |

The most employed PID design technique used in the industry is the Ziegler–Nichols method [1], which avoids the need for a model of the plant to be controlled and relies solely on the step response of the plant. The parameter setting, according to the Ziegler–Nichols method, is carried out in four steps.

1) Obtain the plant step response.
2) Draw the steepest straight-line tangent to the response.
3) Obtain the measures $a$ and $L$, as shown in Fig. 1.
4) Set the parameters $K_p, T_i$, and $T_d$ according to Table I.

It is well known that feedback systems with PID controllers tuned according to the Ziegler–Nichols step response method has good disturbance rejection. However, the compensated system response to a step signal has, in general, a high-percent overshoot, and the control signal is usually high, which may lead the actuator to saturation. In several processes (such as chemical process), high-percent overshoot is not a problem, providing that the system returns rapidly to the neighborhood of the steady-state value. However, in other processes (such as in the manufacture of plastic gloves[1]), it is desirable to have no

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[1]In the process of manufacturing plastic gloves, the positioning of a double plastic film is necessary; thus, if an overshoot occurs, the plastic films wrinkle.

overshoot at all; therefore, the Ziegler–Nichols method cannot be used to tune PID controllers for these systems. Another restriction of Ziegler–Nichols is that it is only suitable for systems with monotonic step response (S-shape response).

More recently, there has been a renewed interest in PID control, and the number of publications in the last ten years has overcome the total number of papers published before 1990 [4]. New tuning rules based on time-domain specifications have been proposed (e.g., [5], [6] and the references therein). Hang *et al.* [5] have reexamined the Ziegler–Nichols method and proposed new tuning formulas for proportional-integral (PI) controllers and the introduction of a setting-point weight for systems with PID controllers. Zhuang and Atherton [6] propose an optimal design of PID controllers based on the minimization of an integral criterion [integral of the square of the product of time and error (ISTE)]. The main purpose of such approaches was to reduce the excessive overshoot of systems compensated with Ziegler–Nichols controllers. The main drawbacks of these methodologies are: 1) no constraint is made on the maximum value for the response overshoot; and 2) like the Ziegler–Nichols method, they are suitable only for plants with monotonic step response.

In this paper, tuning methods for PI and PID controllers are proposed such that the response of the compensated system has overshoot below a prescribed value. The methodology also encompasses the design of PID controllers for plants with underdamped step response. All the development of the methodologies is simple and relies solely on concepts introduced in a frequency-domain-based control course. For this reason, the paper also has a didactic contribution.

This paper is organized as follows. The tuning of PI and PID controllers for plants with monotonic step response is considered in Section II, and the design of PID controllers for plants with underdamped step response is carried out in Section III. Examples are given in Sections II and III to illustrate the proposed strategies. Finally, conclusions are drawn in Section IV.

## II. Design of PI and PID Controllers for Plants with Monotonic Step Response

Initially, the design of PI and PID controllers for plants with monotonic step response are considered. These systems are better modeled by second-order systems (with or without delay). Systems with monotonic step response whose derivative is clearly different from zero at the vicinity of the time instant when the step change occurs should be modeled as first-order systems. Immediately, one can see that, in this case, the values of $a$ and $L$ of Fig. 1 are both zero, precluding the use of the Ziegler–Nichols method. When this situation happens, the design of PI and PID controllers is straightforward [3], [7], [8].

Let $G(s)$ denote the transfer function model of the plant to be controlled, and assume that the system has a step response with the same shape as that shown in Fig. 2. Then $G(s)$ may be modeled as [9]

$$G(s) = \frac{K}{(\tau s + 1)^2}. \quad (3)$$

The parameter $K$ can be computed as follows [3].

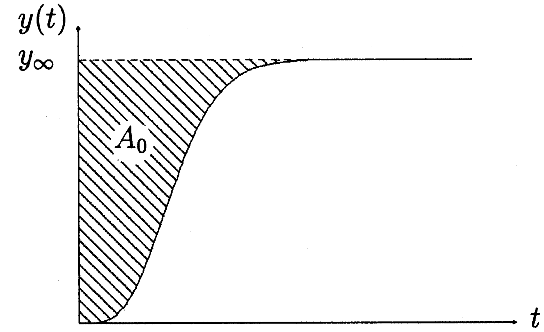


Fig. 2. Step response for the identification of the plant transfer function.

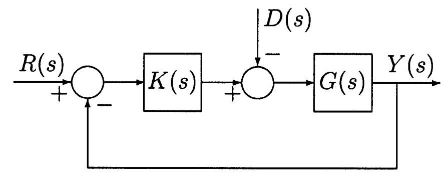


Fig. 3. Block diagram of a feedback control systems.

1) Apply to the plant a step signal with amplitude $A$, and record the response $y(t)$, as shown in Fig. 2.
2) Compute numerically the area $A_0$ and the steady-state value $y_\infty$ of $y(t)$.
3) Compute $K = y_\infty/A$.

For the computation of $\tau$, one case see that for an ideal, critically damped second-order system with transfer function (3), the response to a step with amplitude $A$ is given by [9], as follows:

$$y(t) = KA\left(1 - \frac{1}{\tau}te^{-\frac{1}{\tau}t} - e^{-\frac{1}{\tau}t}\right), \quad t \geq 0.$$

Therefore, since $y_\infty = KA$, then

$$A_0 = \int_0^\infty [KA - y(t)]\,dt = KA\int_0^\infty \left(\frac{1}{\tau}te^{-\frac{1}{\tau}t} + e^{-\frac{1}{\tau}t}\right)dt.$$

After some straightforward calculation, one can check to see that $A_0 = 2KA\tau$, and thus

$$\tau = \frac{A_0}{2y_\infty}. \quad (4)$$

### A. Design of a PI Controller

Consider the feedback system of Fig. 3 and suppose that $G(s)$ is given by (3). When the controller to be designed is a PI, the derivative time $T_d$ is made equal to zero; therefore, (2) assumes the form

$$K(s) = K_p\left(1 + \frac{1}{T_i s}\right) \quad (5)$$

or, equivalently,

$$K(s) = K_p\frac{s + 1/T_i}{s} = K_p\frac{s+z}{s} \quad (6)$$

where $z = 1/T_i$. The problem of setting the PI controller parameters of a critically damped second-order system can then be stated as follows: find a gain $K_p$ and place the zero $-1/T_i$

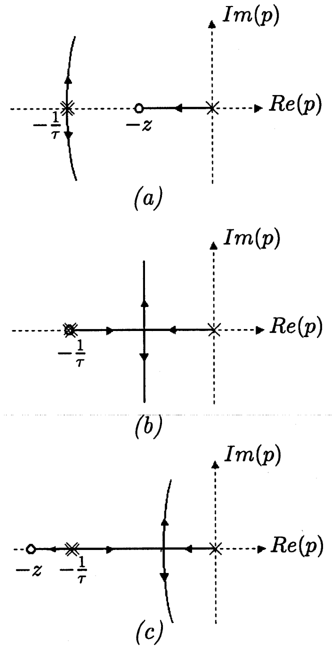


Fig. 4. Root-locus diagram for the design of PI controller. (a) $T_i > \tau$. (b) $T_i = \tau e$. (c) $T_i < \tau$.

such that the feedback system satisfies some transient performance specification. The solution to this problem can be found with the help of the root-locus diagrams of Fig. 4.

1) When the zero $-1/T_i$ is placed between the origin and the double poles $-1/\tau$, i.e., $T_i > \tau$, [Fig. 4(a)], the feedback system step response will be underdamped, which is undesirable because, as far as overshoot is concerned, not all transient performance specifications will be met.
2) If the zero is placed over the double poles i.e., $T_i = \tau$ [Fig. 4(b)], then $K_p$ can be set in such a way that the step response of the closed-loop system may be monotonic or underdamped.
3) When the zero is placed on the left of $-1/\tau$, it is possible to set $K_p$ in order for the feedback system to have either underdamped or monotonic step response, as in the previous case; however, since the root-locus diagram usually repels the poles, then when $K_p$ increases, the closed-loop poles will get closer to the imaginary axis, as shown in Fig. 4(c). Thus, the feedback system will have worse relative stability margins than that obtained in 2) and also a larger settling time.

Therefore, the controller zero must equal $-1/\tau$ or, equivalently, $T_i = \tau$. The choice $T_i = \tau$ for the integral time implies that the closed-loop system will have a second-order transfer function with no poles, namely

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where $\omega_n = \sqrt{KK_p}/\tau$ and $\zeta = 1/\left(2\sqrt{KK_p}\right)$. The implication is that the transient response to a step reference signal will depend on the choice of $K_p$. It is easy to show that for $K_p = 1/(4K)$, the feedback system will be critically damped; whereas for $K_p$ greater (smaller) than $1/(4K)$, the closed-loop system will be underdamped (overdamped). In addition, by replacing $\zeta = 1/\left(2\sqrt{KK_p}\right)$ in the expression for the percent overshoot of a second-order system [9]

$$\mathrm{PO}(\%) = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100\%$$

it is not difficult to obtain the following relationship between $K, \delta = \mathrm{PO}(\%)/100$ and $K_p$:

$$K_p = \frac{1}{4K}\left[1 + \left(\frac{\pi}{\ln \delta}\right)^2\right]. \tag{7}$$

At this point, it is important to note that in this development, it is assumed that the plant to be controlled is modeled exactly as a critically damped second-order system. However, in practice, the system under consideration is not necessarily an exact critically damped second-order system; therefore, the transfer function model (3) is only an approximation for the actual plant. The implication is that the transient performance indexes for this case will not be exactly those used to set $K_p$ in (7); the closer the model step response is to the actual system response, the more accurate will be the setting as far as overshoot is concerned. Thus, a final adjustment on $K_p$ is required in order for the compensated system to meet the performance specifications.

Thus, the tuning of parameters $K_p$ and $T_i$ of a PI controller for a system with a monotonic step response can be carried out according to the following algorithm:

**Algorithm 1**

*1. Apply to the plant a step of amplitude $A$ and record the output $y(t)$.*

*2. Compute $y_\infty$ [the steady-state value of $y(t)$] and the area $A_0$ of Fig. 2.*

*3. Set the integral time as $T_i = A_0/(2y_\infty)$.*

*4. Set initially the proportional gain $K_p$ as follows: a) $K_p = A/(4y_\infty)$ for a critically damped step response; and b) $K_p = A[1 + (\pi/\ln\delta)^2]/(4y_\infty)$ for a percent overshoot equal to $\delta \times 100\%$.*

*5. With the controller embedded in the real system, increase or decrease $K_p$ in order to change the transient response of the compensated system with the view to meeting transient performance specifications.*

## B. *Design of a PID Controller*

The problem of setting the parameters of a PID controller, as in the case of tuning PI controllers studied previously, can be turned into a problem of placing the open-loop zeros of a compensated system. In order to do so, (2) can be rewritten as

$$K(s) = \bar{K}_p \frac{(s+z_1)(s+z_2)}{s} \tag{8}$$

where $-z_1$ and $-z_2(|z_2| > |z_1|$, by assumption) are the controller zeros and

$$\bar{K}_p = K_p T_d, \quad z_1 + z_2 = \frac{1}{T_d}, \quad z_1 z_2 = \frac{1}{T_i T_d}. \tag{9}$$

This discovery implies that the correct setting of parameters $K_p, T_i$, and $T_d$ depends on the choice of $\bar{K}_p, z_1$, and $z_2$. This choice can be made with the help of the root-locus diagram of Fig. 5. It should be noted that since the open-loop transfer function

$$Q(s) = G(s)K(s) = \frac{K\bar{K}_p(s+z_1)(s+z_2)}{s(\tau s+1)^2}$$

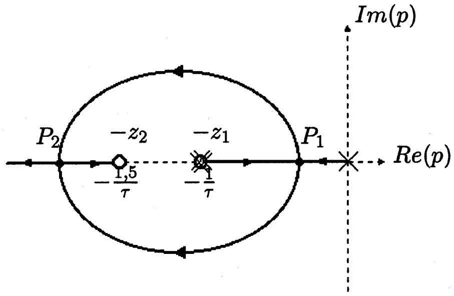


Fig. 5. Root-locus diagram for the tuning of PID controllers.

has a pole at the origin and a double pole at $-1/\tau$, then an immediate choice for $z_1$ is $1/\tau$. This choice will make the closed-loop system behave as a second-order system, and thus, the choice of the closed-loop poles will be easier. In addition, by placing the zero $-z_2$ on the left of $-z_1$, the root-locus diagram will deviate toward $-z_2$ and, therefore, away from the imaginary axis. This condition will improve both the transient response and the relative stability margins.

The choice of the zero $-z_2$ is, therefore, the next step on the setting of PID controllers. Based on several simulation exercises, it has been concluded that the best choice is $z_2 = 1.5/\tau$, and therefore, the open-loop transfer function will be given as

$$Q(s) = \frac{\bar{K}\bar{K}_p(s+z_2)}{s(s+1/\tau)}$$

where $\bar{K} = K/\tau^2$. It can be seen from the root-locus diagram of Fig. 5 that the choice of $\bar{K}_p$ can make the closed-loop system have either damped or underdamped step response. It is not difficult to check that the closed-loop polynomial is given by

$$p_c(s) = s(s+1/\tau) + \bar{K}\bar{K}_p(s+z_2)$$

and thus, $p_c(s) = 0$ will have double roots when

$$\bar{K}_p = \frac{1}{\bar{K}}\left[2\left(z_2 \pm \sqrt{z_2^2 - \frac{z_2}{\tau}}\right) - \frac{1}{\tau}\right]. \tag{10}$$

It should be noted that the smaller (larger) value of $\bar{K}_p$ corresponds to the gain for point $P_1(P_2)$ of the root-locus diagram of Fig. 5. The smaller value of $\bar{K}_p$ will be adopted, since it leads to a smaller control signal. Moreover, since the plants to be controlled do not necessarily have the model given by (3), a high gain may be undesirable from the stability point of view because it may lead to violation of the Nyquist stability criterion at high frequencies, making the actual feedback system unstable.

Finally, noting that $z_1 = 1/\tau, z_2 = 1.5/\tau$, and $\bar{K} = K/\tau^2$, and substituting these expressions in (9) and (10), the parameters $K_p, T_i$, and $T_d$ of the PID controller can be expressed in terms of the plant parameters $K$ and $\tau$ as

$$K_p = \frac{0.6699}{K}, \quad T_i = \frac{5\tau}{3}, \quad T_d = \frac{2\tau}{5}.$$

Therefore, the tune of a PID controller for a plant with montonic step response can be carried out as follows:

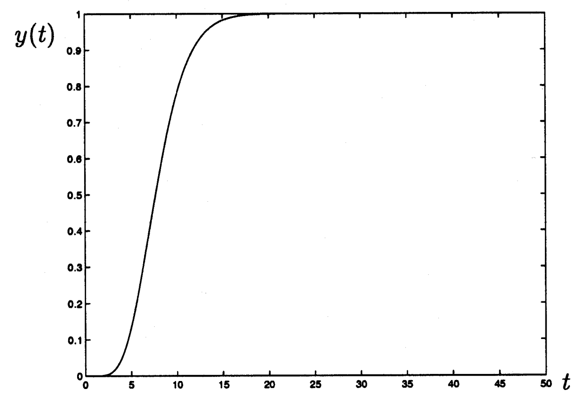


Fig. 6. Unit step response of $G(s) = 1/(s+1)^8$.

TABLE II
PI AND PID PARAMETERS FOR THE PLANT $G(s) = 1/(s+1)^8$

| Method | Controller | | $K_p$ | $T_i$ | $T_d$ |
|---|---|---|---|---|---|
| Proposed | PI | ($\delta = 0$) | 0.25 | 4.00 | |
| | | ($\delta = 0.05$) | 0.5249 | 4.00 | |
| | PID | ($\delta = 0$) | 0.6699 | 6.6667 | 1.6 |
| | | ($\delta = 0.05$) | 0.8460 | 6.6667 | 1.6 |
| Ziegler-Nichols | PI | | 1.4025 | 12.9205 | |
| | PID | | 1.8699 | 8.6137 | 2.1534 |

**Algorithm 2**

*1. Apply to the plant a step signal of amplitude $A$ and record the output $y(t)$.*

*2. Compute $y_\infty$ [the steady-state value of $y(t)$] and the area $A_0$ of Fig. 2.*

*3. Set the controller parameters as $K_p = 0.6699/K, T_i = 5A_0/(6y_\infty)$, and $T_d = A_0/(5y_\infty)$.*

*4. With the controller embedded in the real system, increase or decrease $K_p$ in order to change the transient response of the compensated system with the view to meeting the performance specifications.*

### *C. Example*

The results of the methodology proposed in this section will be illustrated by the design of PI and PID controllers for the following plant [3] (assumed unknown):

$$G(s) = \frac{1}{(s+1)^8}.$$

The plant response to a unit-step input is shown in Fig. 6, from which it can be seen that $y_\infty = 1$, and thus, $K = 1$. In addition, according to Algorithms 1 and 2, all the parameters necessary to tune PI and PID controllers for this plant can be obtained from Fig. 6. For the tuning of PI and PID controllers according to the Ziegler–Nichols method, it is necessary to find $a$ and $L$, which according to Fig. 6, are equal to 0.6417 and 4.3068, respectively. In order to set the parameters in accordance with Algorithms 1 and 2, it is necessary to compute numerically the area $A_0$, which is equal to 8.0. The parameters of PI and PID controllers adjusted via Ziegler–Nichols and Algorithms 1 and 2 are given in Table II, and the corresponding responses of the closed-loop systems to a unit-step reference signal are shown in Fig. 7

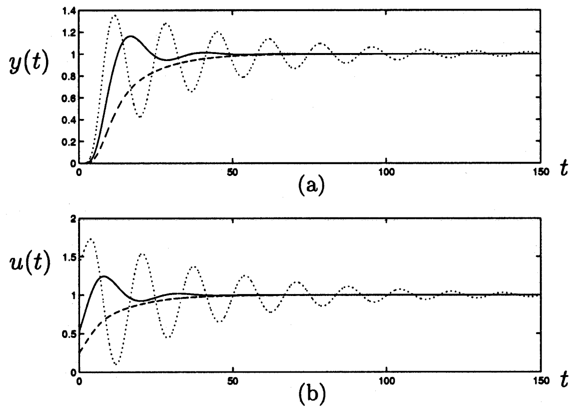


Fig. 7. (a) Step response and (b) control signal for the closed-loop system with the PI controllers of Table II. Ziegler–Nichols (dotted lines). Proposed methodology with $K_p = 0.25$ (dashed lines). $K_p = 0.5249$ (solid lines).

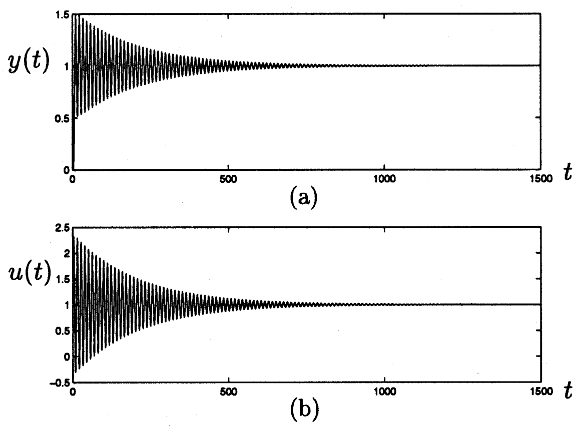


Fig. 8. (a) Step response and (b) control signal for the closed-loop system with the Ziegler–Nichols PID controller given in Table II.

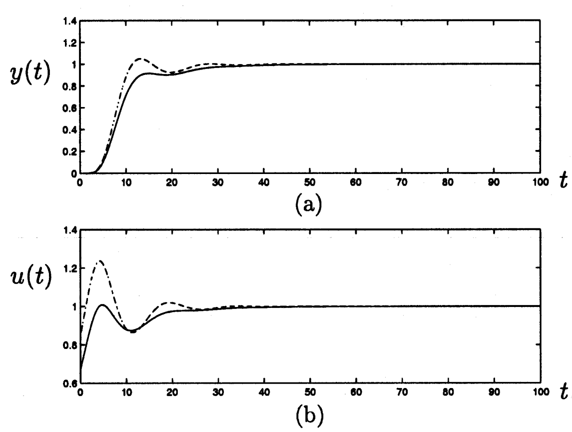


Fig. 9. (a) Step response and (b) control signal for the closed-loop system with PID controllers tuned by following the proposed methodology with $K_p = 0.6699$ (solid lines) and $K_p = 0.8460$ (dashed lines).

(closed-loop system with PI controllers) and Figs. 8 and 9 (closed-loop system with PID controllers). The performance indexes are given in Table III, where $t_r, t_p$, and $t_s$ denote the rise time, peak time, and 2% settling time, respectively; PO(%) is the percent overshoot; and $u_{\max}$ is the maximum absolute

TABLE III
TRANSIENT PERFORMANCE INDEXES FOR THE COMPENSATED SYSTEM WITH PI AND PID CONTROLLERS OF TABLE II

| Method | Controller | | $t_r$ | $t_p$ | $t_s$ | PO(%) | $u_{\max}$ |
|---|---|---|---|---|---|---|---|
| Proposed | PI | $(\delta = 0)$ | 24.3 | — | 49.8 | — | 1.0 |
| | | $(\delta = 0.05)$ | 12.6 | 16.8 | 34.0 | 16.4 | 1.2 |
| | PID | $(\delta = 0)$ | 8.3 | — | 33.8 | — | 1.0 |
| | | $(\delta = 0.05)$ | 11.4 | 13.2 | 24.7 | 5.0 | 1.2 |
| Ziegler-Nichols | PI | | 8.6 | 11.6 | 138.8 | 35.4 | 1.7 |
| | PID | | 7.5 | 22.1 | 678.3 | 49.7 | 2.3 |

value of the control signal. Notice that in Fig. 7, the unit-step response of the closed-loop system with a Ziegler–Nichols PI controller (dotted line) is more oscillatory, has a higher percent overshoot, and a larger settling time than those of the feedback systems with PI compensators tuned in accordance with Algorithm 1.

In addition, Algorithm 1 allows the designer to vary the proportional gain in a systematic way, accelerating the system responses. When PID controllers are used as compensators, the performance of the system with PID tuned with the proposed methodology is also superior to that with a Ziegler–Nichols compensator. The unit-step response of a closed-loop system with a PID controller tuned in accordance with Ziegler–Nichols (see Table II, bottom row) is shown in Fig. 8. It can been seen that this response is highly oscillatory, has a high-percent overshoot ($\approx$ 50 %), and has an extremely large settling time (see Table III, bottom row). It is, therefore, unacceptable. On the other hand, the same feedback system with PID controllers tuned according to Algorithm 2 have unit-step responses with little oscillation, low-percent (or no) overshoot, and short settling time, as can be seen from Fig. 9 and in rows 3 and 4 of Table III. As in the case of PI controllers, the settling time of the closed-loop response can also be made smaller by allowing a larger overshoot. For instance, according to Table III, for $\delta = 0.05$, the settling time is 24.7 s, but for $\delta = 0$, it is 33.8 s.

## III. DESIGN OF PID CONTROLLERS FOR PLANTS WITH UNDERDAMPED STEP RESPONSE

Systems with underdamped step response may be approximated by a second-order system with the transfer function [9]

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{11}$$

where $0 < \zeta < 1$. For an ideal second-order system with transfer function (11), the response to a step input with amplitude $A$ is given as [9]

$$y(t) = KA\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}t + \phi\right)\right] \tag{12}$$

toward appropriately tuning the controller gain.where $\phi = \arccos(\zeta)$. Thus the dc-gain $K$, the damping coefficient $\zeta$ and the natural frequency $\omega_n$ can be determined as follows [3].

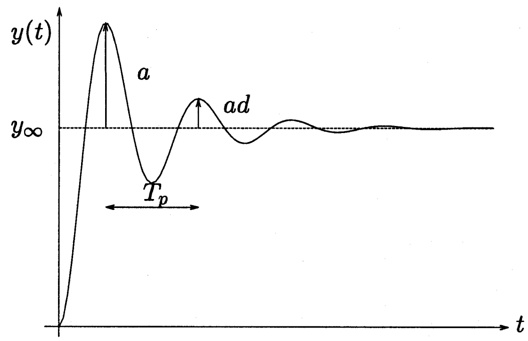


Fig. 10. Typical step response of an underdamped second-order system.

1) Apply a step signal with amplitude $A$ and record the output $y(t)$.
2) Let $y_\infty$ denote the steady-state value of $y(t)$ and find the oscillation period $T_p$ and the decay ratio $d$ given in Fig. 10.
3) Compute $\zeta$ and $\omega_n$ as follows:

$$\zeta = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\ln d}\right)^2}}, \quad \omega_n = \frac{2\pi}{T_p\sqrt{1-\zeta^2}}.$$

In order to obtain a PID controller for this system, it should be noted that (2) can be rewritten as

$$K(s) = \frac{K_p T_d}{s}\left(s^2 + \frac{1}{T_d}s + \frac{1}{T_i T_d}\right). \tag{13}$$

Since system (11) has a pair of complex poles, namely $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$, a natural choice for the controller zeros would be such that the numerator polynomial of $K(s)$ and the denominator polynomial of $G(s)$ cancel. In order for this condition to happen, $T_d$ and $T_i$ must satisfy

$$T_d = \frac{1}{2\zeta\omega_n}, \quad T_i = \frac{2\zeta}{\omega_n}. \tag{14}$$

This choice for $T_d$ and $T_i$ would make the closed-loop system behave exactly as a first-order system with the transfer function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{1}{\bar{\tau}s + 1}$$

where $\bar{\tau} = T_i/(K_p K)$. The relationship between the settling time $(t_s)$ and the time constant $(\bar{\tau})$ of a first-order system is $t_s = 4\bar{\tau}$, and, therefore, for a given settling time $t_s$, the controller gain should be adjusted to $K_p = 4T_i/(Kt_s)$. However, in practice, the closed-loop system will never behave as a first-order one because the plant is not exactly a second-order system and also because, in the actual controller, the derivative action is performed not on the error signal but on the output signal. Therefore, this choice of $K_p$ can only serve as a first step.

It is important to note that, in a recent book [3], the problem of designing PID controllers for systems with underdamped step response has been tackled. Indeed, the same choice for $T_d$ and $T_i$ as those given in (14) has been proposed in [3] for a plant model slightly different from (11). However, the problem of choosing the controller gain $K_p$ has not been pursued in [3].

In this paper, a systematic way to tune $K_p$ will be proposed. The idea behind the approach is that since the controller design is based on an approximate model, not all desired settling time will be achieved. Thus, it is recommended to start with a controller that does not considerably change the plant performance. This procedure can be accomplished by choosing $t_s$ approximately equal to the settling time of the step response of the plant to be controlled.

The results of this section can be summarized in the following algorithm:

**Algorithm 3**

*1. Apply to the plant a step of amplitude $A$ and record the output $y(t)$.*
*2. Determine $y_\infty$ [the steady-state value of $y(t)$], the settling time $t_{s_0}$ of the plant response, the first two peak values $M_{p_1}$ and $M_{p_2}$, and the corresponding time instants $t_{p_1}$ and $t_{p_2}$.*
*3. Compute $d = (M_{p_2} - y_\infty)/(M_{p_1} - y_\infty)$ and $T_p = t_{p_2} - t_{p_1}$.*
*4. Compute $\zeta = 1/\sqrt{1+(2\pi/\ln d)^2}$ and $\omega_n = 2\pi/(T_p\sqrt{1-\zeta^2})$.*
*5. Set $T_d = 1/(2\zeta\omega_n)$, $T_i = 2\zeta/\omega_n)$ , and $K_p = 4T_i/(Kt_{s_0})$.*
*6. With the controller embedded in the real system, increase $K_p$ up to a value for which either the settling time starts to increase again or the percent overshoot becomes higher than a prescribed value.*

### A. Example

To illustrate the methodology proposed in this section, one may consider the problem of finding a PID controller for a plant with the (assumed unknown) transfer function found in (15) at the bottom of the page. This transfer function is not associated with any physical process and was chosen at random with the restrictions to be stable and to have dominant complex conjugate poles.

According to Algorithm 3, the first step for the tuning of PID controllers is to obtain the plant step response. For this plant, the unit-step response is shown in Fig. 11, from which it is possible to verify that $t_{s_0} = 21.2$ s, $y_\infty = 1.28, M_{p_1} = 1.40, M_{p_2} = 1.34, t_{p_1} = 9.05$ s, and $t_{p_2} = 15.45$ s. Following steps 3 and 4 of Algorithm 3, one obtains $K = 1.2857, \zeta = 0.1169$, and $\omega_n = 0.9885$. Therefore, according to step 5, the controller integral and derivative times are $T_i = 0.2366$ and $T_d = 4.3251$, respectively. The controller gain adjustment is carried out by using the plant settling time ($t_{s_0} = 21.2$ s) as the initial choice for the desired settling time of the closed-loop system. Thus,

$$G(s) = \frac{28.50s^2 + 6.93s + 18.20}{s^6 + 17.47s^5 + 46.78s^4 + 67.52s^3 + 64.86s^2 + 43.30s + 14.16}. \tag{15}$$

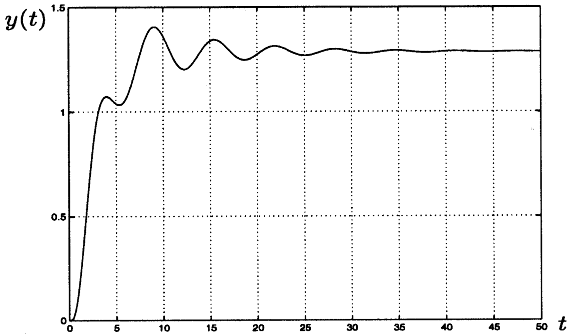


Fig. 11. Unit-step response of the plant of Section III-A.

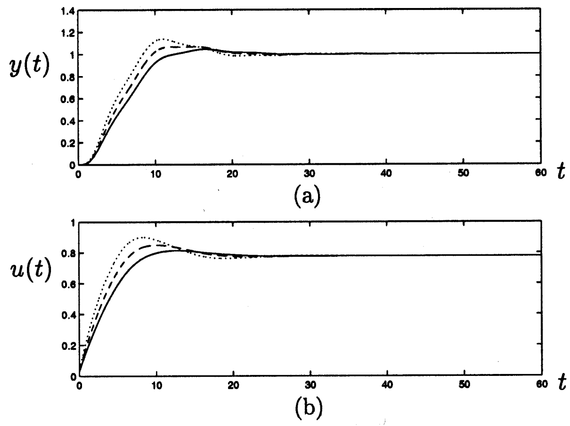


Fig. 12. Step response (a) and control signal (b) for the closed-loop system for $K_p = 0.0033$ (solid lines), $K_p = 0.0399$ (dashed lines), and $K_p = 0.0482$ (dotted lines).

TABLE IV
TRANSIENT PERFORMANCE INDEXES FOR THE CLOSED-LOOP SYSTEM WITH PID PARAMETERS $T_i = 0.2366$ and $T_d = 4.3251$ WITH DIFFERENT VALUES OF GAIN $K_p$

| $K_p$ | $t_r$ | $t_p$ | $t_s$ | P.O. (%) |
|---|---|---|---|---|
| 0.0333 | $12.5s$ | $16.5s$ | $19.7s$ | 4.4 |
| 0.0399 | $9.65s$ | $15.3s$ | $19.1s$ | 6.7 |
| 0.0482 | $8.44s$ | $11.0s$ | $17.9s$ | 13.7 |
| 0.0499 | $8.26s$ | $10.9s$ | $21.0s$ | 15.0 |

$K_{p_0} = 0.0333$ is the initial value for the controller gain. The closed-loop response to a unit-step reference signal is shown in Fig. 12 (solid lines), and the corresponding transient performance indexes $t_r, t_p, t_s$, and PO are given in Table IV. It should be noted that for $K_p = 0.0333$, the closed-loop response has a small percent overshoot (4.4%), and the actual settling time (19.7 s) is smaller than the desired one (22.1 s). In the sequel, by increasing the gain $K_p$, one can see from Fig. 12 and Table IV that, as predicted in Algorithm 3, the settling time of the closed-loop response decreases up to approximately 17.9 s (for $K_p = 0.0482$) and then starts to increase again. Thus, $K_p = 0.0482$ can be adopted as the controller gain if the designer is concerned with the speed of the system to reach steady state.

## IV. CONCLUSION

In this paper, methodologies for tuning PI and PID controllers have been proposed. Like the well-known Ziegler–Nichols method, they are based on the plant step response. Unlike the Ziegler–Nichols step response method, they provide systematic means to adjust the proportional gain in order to have no overshoot on the closed-loop step response. In addition, PID controllers can be designed for plants with underdamped step response. Examples are given to illustrate the efficiency of the methodology. The paper also provides a didactic contribution since the proposed methodology is based on root-locus diagrams and, therefore, can be used in an undergraduate control course.

## REFERENCES

[1] J. G. Ziegler and N. B. Nichols, "Optimal settings for automatic controllers," *Trans. ASME*, vol. 64, pp. 759–768, 1942.
[2] K. J. Astrom and T. Haglund, *Automatic Tuning of PID Controllers*, 1st ed. Research Triangle Park, NC: Instrum. Soc. Amer., 1988.
[3] K. J. Astrom and T. Haglund, *PID Controllers: Theory, Design and Tuning*, 2nd ed. Research Triangle Park, NC: Instrum. Soc. Amer., 1995.
[4] S. Bennett, "The past of PID controllers," in *Proc. IFAC Workshop on Digital Control: Past, Present and Future of PID Control*, Terrassa, Spain, 2000, pp. 1–11.
[5] C. C. Hang, K. J. Astrom, and W. K. Ho, "Refinements of the Ziegler–Nichols tuning formula," *Proc Inst. Elect. Eng.—Part D: Control Theory and Applications*, vol. 138, no. 2, pp. 111–118, 1991.
[6] M. Zhuang and D. P. Atherton, "Automatic tuning of optimum PID controllers," *IEE Proceedings—Part D: Control Theory and Applications*, vol. 140, no. 3, pp. 216–224, 1993.
[7] J. C. Basilio, "A laboratory for a first course in control systems," *Int. J. Elect. Eng. Educ.*, no. 39, pp. 54–70, 2002.
[8] G. K. I. Mann, B. G. Hu, and R. G. Gosine, "Time-domain based design and analysis of new PID tuning rules," *Proc. Inst. Elect. Eng.—Control Theory and Applications*, vol. 148, no. 3, pp. 251–261, 2001.
[9] K. Ogata, *Modern Control Engineering*, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 1997.

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